Honors Precalculus – Continuity　　　　　　　　　Name: ______________________________
Review exercises – from *Calculus*, Finney et al

Section 2.3 Exercises

In Exercises 1–10, find the points of discontinuity of the function. Identify each type of discontinuity.

1. $y = \dfrac{1}{(x+2)^2}$

2. $y = \dfrac{x+1}{x^2 - 4x + 3}$

3. $y = \dfrac{1}{x^2 + 1}$

4. $y = |x - 1|$

5. $y = \sqrt{2x + 3}$

6. $y = \sqrt[3]{2x - 1}$

7. $y = |x|/x$

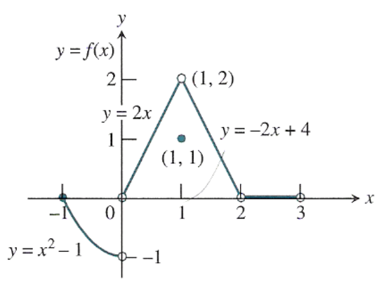
8. $y = \cot x$

9. $y = e^{1/x}$

10. $y = \ln (x + 1)$

In Exercises 11–18, use the function f defined and graphed below to answer the questions.

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$



11. **(a)** Does $f(-1)$ exist?

(b) Does $\lim_{x\to -1^+} f(x)$ exist?

(c) Does $\lim_{x\to -1^+} f(x) = f(-1)$?

(d) Is f continuous at $x = -1$?

12. **(a)** Does $f(1)$ exist?

(b) Does $\lim_{x\to 1} f(x)$ exist?

(c) Does $\lim_{x\to 1} f(x) = f(1)$?

(d) Is f continuous at $x = 1$?

13. **(a)** Is f defined at $x = 2$? (Look at the definition of f.)

(b) Is f continuous at $x = 2$?

14. At what values of x is f continuous?

15. What value should be assigned to $f(2)$ to make the extended function continuous at $x = 2$?

16. What new value should be assigned to $f(1)$ to make the new function continuous at $x = 1$?

17. **Writing to Learn** Is it possible to extend f to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

18. **Writing to Learn** Is it possible to extend f to be continuous at $x = 3$? If so, what value should the extended function have there? If not, why not?

In Exercises 19–24, **(a)** find each point of discontinuity. **(b)** Which of the discontinuities are removable? not removable? Give reasons for your answers.

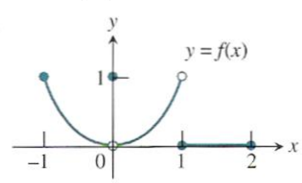
19. $f(x) = \begin{cases} 3 - x, & x < 2 \\ \dfrac{x}{2} + 1, & x > 2 \end{cases}$

20. $f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ x/2, & x > 2 \end{cases}$

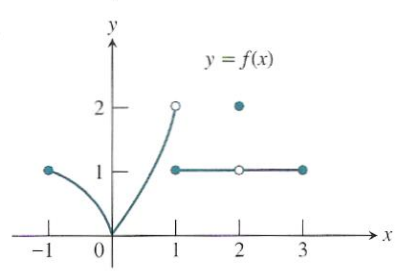
21. $f(x) = \begin{cases} \dfrac{1}{x - 1}, & x < 1 \\ x^3 - 2x + 5, & x \ge 1 \end{cases}$

22. $f(x) = \begin{cases} 1 - x^2, & x \ne -1 \\ 2, & x = -1 \end{cases}$

23.



24.



In Exercises 25–30, give a formula for the extended function that is continuous at the indicated point.

25. $f(x) = \dfrac{x^2 - 9}{x + 3}, \quad x = -3$

26. $f(x) = \dfrac{x^3 - 1}{x^2 - 1}, \quad x = 1$

27. $f(x) = \dfrac{\sin x}{x}, \quad x = 0$

28. $f(x) = \dfrac{\sin 4x}{x}, \quad x = 0$

29. $f(x) = \dfrac{x - 4}{\sqrt{x} - 2}, \quad x = 4$

30. $f(x) = \dfrac{x^3 - 4x^2 - 11x + 30}{x^2 - 4}, \quad x = 2$

For each of the following, find a value of *a* so that the function is continuous:

31. $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$

32. $f(x) = \begin{cases} 2x + 3, & x \leq 2 \\ ax + 1, & x > 2 \end{cases}$

33. $f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \geq -1 \end{cases}$

34. $f(x) = \begin{cases} x^2 + x + a, & x < 1 \\ x^3, & x \geq 1 \end{cases}$

Section 2.2 Exercises

Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$:

1. $f(x) = \cos\left(\frac{1}{x}\right)$

2. $f(x) = \frac{\sin 2x}{x}$

23. $y = \frac{\cos(1/x)}{1 + (1/x)}$

24. $y = \frac{2x + \sin x}{x}$

25. $y = \frac{\sin x}{2x^2 + x}$

Find the limits:

19. $\lim_{x \to 0^+} \csc x$

20. $\lim_{x \to (\pi/2)^+} \sec x$

In Exercises 9–12, find the limit and confirm your answer using the Sandwich Theorem.

9. $\lim_{x \to \infty} \frac{1 - \cos x}{x^2}$

10. $\lim_{x \to -\infty} \frac{1 - \cos x}{x^2}$

11. $\lim_{x \to -\infty} \frac{\sin x}{x}$

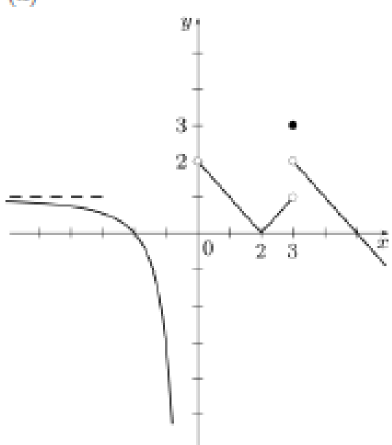
12. $\lim_{x \to \infty} \frac{\sin(x^2)}{x}$

201-103-RE - Calculus 1

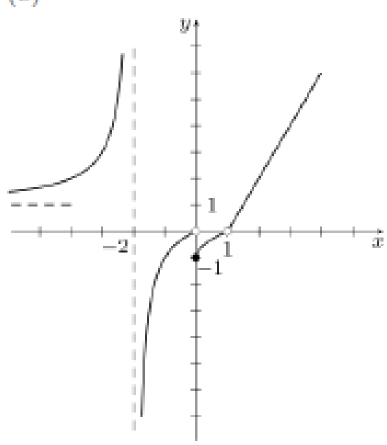
WORKSHEET: CONTINUITY

1. For each graph, determine where the function is **discontinuous**. Justify for each point by: (i) saying which condition fails in the definition of continuity, and (ii) by mentioning which type of discontinuity it is.

(a)



(b)



2. For each function, determine the interval(s) of continuity.

   (a) $f(x) = x^2 + e^x$

   (b) $f(x) = \dfrac{3x+1}{2x^2 - 3x - 2}$

   (c) $f(x) = \sqrt[4]{5-x}$

   (d)* $f(x) = \dfrac{2}{4-x^2} + \dfrac{1}{\sqrt{x^2 - x - 12}}$

3. For each piecewise defined function, determine where $f(x)$ is continuous (or where it is discontinuous). Justify your answer in detail.

(a) $f(x) = \begin{cases} 2^x - 3x^2 & \text{for } x \leq 1 \\ \log_{10}(x) + x & \text{for } x > 1 \end{cases}$

(b) $f(x) = \begin{cases} \frac{2x}{3-x} & \text{for } x \leq 0 \\ x^2 - 3x & \text{for } 0 < x < 2 \\ \frac{x^2-8}{x} & \text{for } x > 2 \end{cases}$

4. Find all the value(s) of the parameter c (if possible), to make the given function continuous everywhere.

   (a) $f(x) = \begin{cases} c \cdot 3^x - x^2 + 2c & \text{for } x \leq 0 \\ 2x^5 + c(x+1) + 16 & \text{for } x > 0 \end{cases}$

   (b) $f(x) = \begin{cases} 2(cx)^3 + x - 1 & \text{for } x \leq 1 \\ 2cx + (x-1)^2 & \text{for } x > 1 \end{cases}$

   (c) $f(x) = \begin{cases} 3x + c & \text{for } x < -1 \\ x^2 - c & \text{for } -1 \leq x \leq 2 \\ 3 & \text{for } x > 2 \end{cases}$

<u>Answers</u> :

Section 2.3: 2-24 even

2. $x=1$ and $x=3$, both infinite discontinuities; 4. none, 6. none, 8. $x=k\pi$ for all integers *k*, infinite discontinuity; 10. all points not in the domain, i.e., all $x<-1$; 12a. yes, 12b. yes, 12c. no, 12d. no, 14. everywhere in $[-1, 3)$ except for $x=0, 1, 2$; 16. 2, 18. yes, assign the value 0 to $f(3)$; 20a. $x=2$, 20b. removable, assign the value 1 to $f(2)$; 22a. $x=-1$. 22b. removable, assign the value 0 to $f(-1)$; 24a. all points not in the domain along with $x=0, 1$; 24b. $x=1$ is not removable, the one-sided limits are different; $x=2$ is a removable discontinuity, assign $f(2)=1$

Section 2.3: 1-23 odd

Exercises 2.3

1. $x=-2$, infinite discontinuity **3.** None **5.** All points not in the domain, i.e., all $x<-\frac{3}{2}$ **7.** $x=0$, jump discontinuity **9.** $x=0$, infinite discontinuity

11. (a) Yes (b) Yes (c) Yes (d) Yes **13.** (a) No (b) No **15.** 0

17. No, because the right-hand and left-hand limits are not the same at zero.

19. (a) $x=2$ (b) Not removable, the one-sided limits are different.

21. (a) $x=1$

(b) Not removable, it's an infinite discontinuity.

23. (a) All points not in the domain along with $x=0, 1$

(b) $x=0$ is a removable discontinuity, assign $f(0)=0$.

$x=1$ is not removable, the two-sided limits are different.

Section 2.2:

1. both are 1, 2. both are 0, 23. both are 1, 24. both are 2, 25. both are 0, 19. ∞, 20. $-\infty$, 9. 0, 10. 0, 11. 0, 12. 0

Worksheet: Continuity

1. (a) $x=0,3$ (b) $x=-2,0,1$
2. (a) $\mathbb{R}$ (b) $\mathbb{R}\backslash\{-1/2,2\}$ (c) $(-\infty,5]$ (d) $(-3,2)\cup(-2,2)\cup(2,4)$
3. (a) discontinuous only at $x=1$ (b) discontinuous only at $x=2$
4. (a) $c=8$ (b) $c=-1,0,1$ (c) no solution possible