Rotational Motion

-*Translational Motion* refers to straight line motion.
-*Rotational Motion* refers to motion about a fixed vertical axis.

ROTATIONAL MOTION

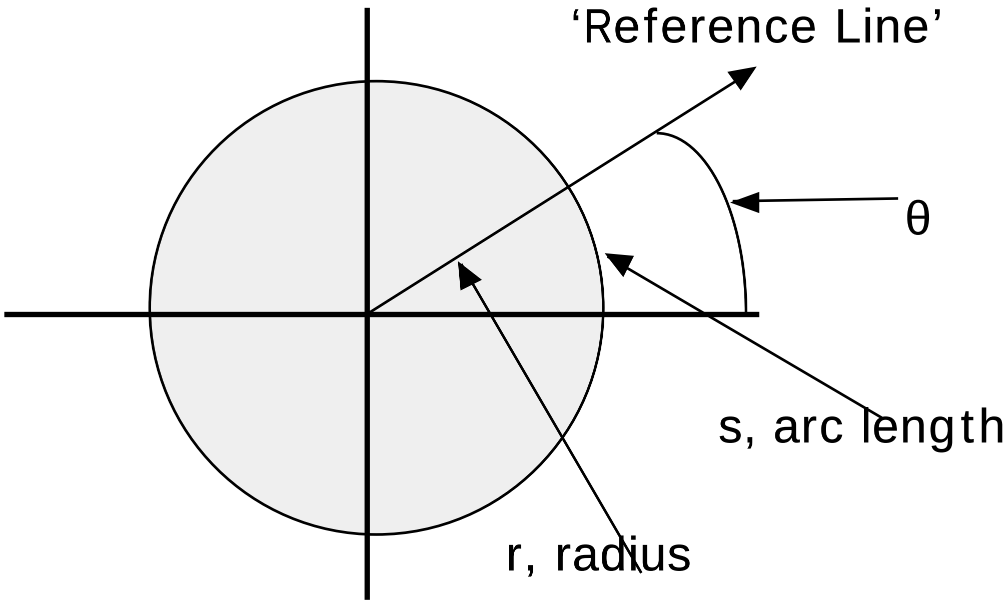
We define rotational motion to be the motion of a rigid body about a fixed axis. Thus, wheels and gears fit this definition, a bowling ball rolling or the rotation of the sun do not. Why?

In rotational, every point on the body moves in a circle about the axis of rotation, and every point has the same <u>angular</u> rotation.

In straight line translational, every point receives the same <u>linear</u> displacement.

We can compare our rotational variables to our translational variables:

Theta, θ, the angular position, comparable to "x",
Omega, ω, the angular velocity, comparable to "v",
Alpha, α, the angular accel., comparable to "a".



The <u>angular position</u> of the reference line is given by:

$$\theta = s/r$$, radian measure

(remember, 2π radians in a circle,1 radian = 57.3º)

The <u>average angular velocity</u> is given by:

$$\omega = \Delta\theta/\Delta t$$

For the <u>instantaneous angular velocity</u> we find the limit as t approaches zero, and :

$$\omega = d\theta/dt$$

Units typically are: radians/second or rev/sec

The <u>average angular acceleration</u> is given by:

$$\alpha = \Delta\omega/\Delta t$$

For the <u>instantaneous angular acceleration</u> we find the limit as t approaches zero, and :

$$\alpha = d\omega/dt$$

Units typically are: radians/second^2 or rev/sec^2

THE CONSTANT ANGULAR ACCELERATION CASE

The equations of motion for angular parallel those for translational when the acceleration is constant:

TRANSLATIONAL	ROTATIONAL
$v_f=v_o + at$	$\omega_f=\omega_o + \alpha t$
$x=v_o t + .5at^2$	$\theta=\omega_o t + .5\alpha t^2$
$v_f^2=v_o^2 + 2ax$	$\omega_f^2=\omega_o^2 + 2\alpha\theta$
$x = .5(v_o + v_f)\ t$	$\theta = .5(\omega_o + \omega_f)\ t$

EXAMPLES:

1. One quarter is placed on a table and not allowed to rotate. A second quarter is placed next to it, "teeth to teeth", and rolled around the first. Through what angle will the second quarter have turned?

720 degrees!!! Once around itself, once around the other quarter.

2. A Grinding wheel has a constant angular accel. of .35 rad/sec2. It starts from rest.

a) What is the angular displacement of an arbitrary reference line from start to 18 seconds?

$\theta=\omega_o t + .5\alpha t^2$

$\theta=0 + .5(.35)(18^2)$

$\theta=57$ rad or 3200^o or 9 revolutions

b) What is the angular speed of the wheel at t=18sec?

$\omega_f=\omega_o + \alpha t$

$\omega_f=0 + .35(18)$

ω_f=6.3 rad/sec

or 6.3 rad/sec times 1 rev/2πrad = 1 rev/sec

3. Using the grindstone of Angular Acceleration .35 rad/sec^2, start with an initial angular velocity of -4.6 rad/sec. At what time will the grindstone momentarily come to a rest?

$\omega_f = \omega_o + \alpha t$

So $t = (\omega - \omega_o) / \alpha$

t = (0 - - 4.6) / .35 = 13 sec

After 32 seconds, how many revolution will the wheel have experienced?

$\theta = \omega_o t + .5\alpha t^2$

$\theta = -4.6(32) + .5(.35)(32^2)$

= 32 rad

÷ 2π = 5.09 revolutions (2 π rad per rev)

LINEAR AND ANGULAR RELATIONSHIPS

¿What is the angular speed of a Physics day student on the inner horses of a carousel versus the outer horses?
What about linear speed?
Other Examples?

The variables are all “connected” by r, the radius.

POSITION: $s = \theta r$, with s the linear position, θ in radians, measured from an arbitrary axis, r the radius

VELOCITY: The Derivative of Position with respect to time is the Velocity.

$$\frac{ds}{dt} = \frac{d\theta}{dt} r$$

or $v = \omega r$ (ω in radian measure)

as before, v is tangent to the circle of rotation.

ACCELERATION: Take Derivative again.

$$\frac{dv}{dt} = \frac{d\omega}{dt} r$$

This gives us the tangential acceleration only!

$a_t = \alpha r$ (α in radians)

From Circular Motion, we also have a radial or centripetal acceleration,

$a_r = v^2 / r$ or $a_r = \omega^2 r$ (radians for ω)

Thus, we need to consider two components to the acceleration. When is the tangential acceleration equal to zero?

Example:

In the classic stupid film, “Spies Like Us”, the leads are subject to accelerations on a centrifuge.

The radius r of a centrifuge is 15 meters.

a) If the most highly trained people can stand an 11g acceleration, at what constant angular velocity should the centrifuge be set?

No tangential accel, only radial! Use: $a_r = \omega^2 r$

$\omega^2 = a_r \div r$

$= (11*10) \div 15$

= 7.33

ω = 2.7 rad/s or 25.7 rpm

b) What is the Linear Speed at this point?

$v = \omega r$

v = 2.7rad/s (15m) = 40.5 m/s

<u>Kinetic Energy and Inertia of Rotation</u>

Translational Kinetic energy is dependent on mass. Rotational? Try spinning an object . What is the force you feel dependent on?

Rotational Kinetic Energy depends on the distribution of the mass! Just as we defined mass as the inertia of an object, we can also define the rotational Inertia of an object.

$I = \Sigma m_i r_i^2$ Which tells us how the mass of the rotating body is distributed about its axis of rotation. "i" refers to each particle of the object.

Rotational inertia is often called the "Moment of Inertia".

Thus, $K_r = 1/2 \; I \omega^2$

Again, rotational inertia depends on not only the mass, but also the distribution of the mass.

Can you think of a way to demonstrate this inertia?
CALCULATING THE ROTATIONAL INERTIA

For a body made up of discrete particles, we can use:

$I = \Sigma\, m_i r_i^2$

But, for a continuous body, we must replace the sum with an integral:

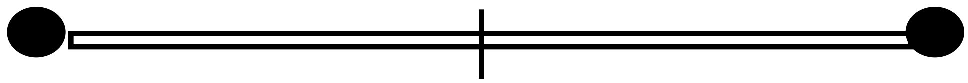
$I = \int r^2 \, dm$

Most Values for rotational inertia are found from tables, such as in the text.

A rather odd but occasional useful theorem, proved in the text, is called the “Parallel Axis Theorem”.
If you want to find the rotational inertia of a body through an axis that is NOT at the center, but is parallel, this is the ticket.

$I = I_{cm} + Mh^2$

Where M is mass of the body, h is the perpendicular distance between the two axis, and I_{cm} is the Inertia at the center point.



EXAMPLES:

1. Two masses are placed at the end of a light (read: Non-existent) rod of length L.

a. What is the rotational inertia of this body about an axis through its center?

$I = \Sigma\, m_i r_i^2$

$I = \Sigma\, (m)(1/2L)^2 \quad + \quad (m)(1/2L)^2$

$= \; 1/2\, mL^2$

b. What is the rotational inertia about an axis through one of the masses?

By Direct Calc:

$I = \Sigma\, m_i r_i^2$

$I = \Sigma\ (m)(0)^2 \ + \ (m)(L)^2$

$= \ mL^2$

By Parallel Axis Theorem:

$I = I_{cm} + Mh^2$

$I = 1/2\ mL^2 \ + 2m(1/2\ L)^2$

$= \ mL^2$ (same as above)

2. Refer to Table 2: Some Rotational Inertia

We have a uniform rod of mass 3 kg and length .8 meters.

a) What is the rotational inertia about its center of mass?

Use: $I = 1/12\ ML^2$

$I = .083\ (3)\ (.8)^2$

$I = .159\ kg \bullet m^2$

b) What is its rotational inertia about its end point?

Method 1: // Axis Theorem

$I = I_{cm} + Mh^2$

$I = .159 + (3)\ (.4)^2$

$I = .639$

Method 2: Use Table

$I = 1/3\ ML^2$

$I = 1/3\ (3)\ (.8)^2$
$I = .64$ (same)

3. A flywheel of mass 75kg has a radius of 25cm. If it is given a velocity of 85,000 rev/min, How much rotational Kinetic Energy can it store? We will assume a cylinder rotating about its axis.

$I = 1/2\ MR^2$

$I = 1/2\ (75kg)\ (.25m)^2$

$I = 2.34\ kg \bullet m^2$

Must find Angular Velocity in rads:
$\omega = 85000$ rev/min $(2\pi$ rad/rev) (1min/60 sec)
= 8900 rad/sec

Then:

$K_r = 1/2\ I\ \omega^2$

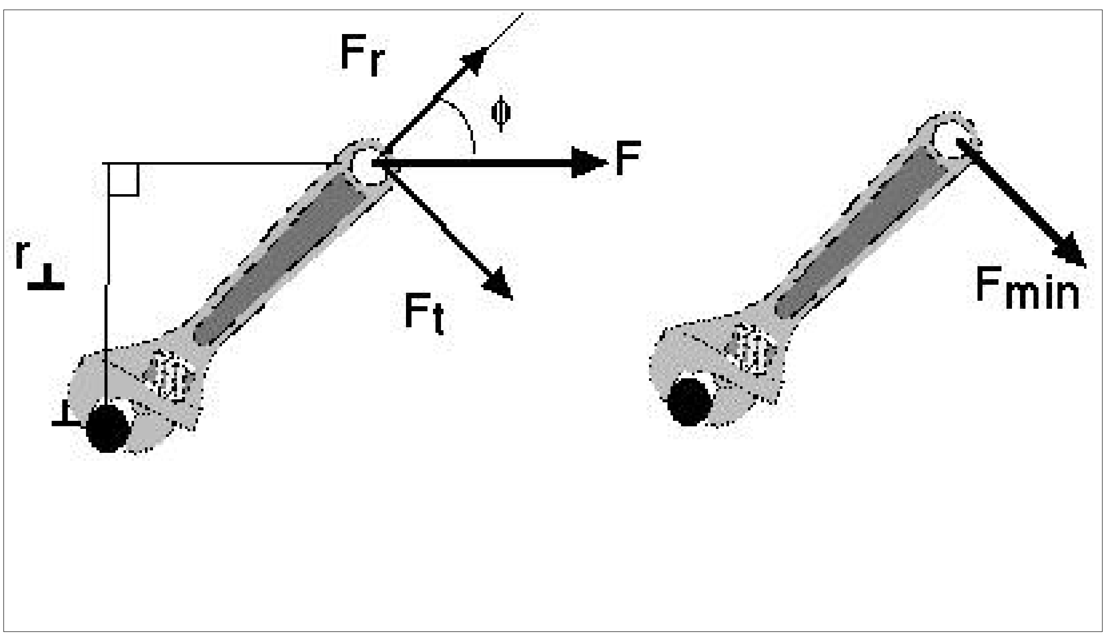
$= 1/2\ (2.34)(8900)^2$

= 9.3 EE 7 $kg \bullet m^2/s^2$ or $N \bullet m$ or J

TORQUE

Just as Rotational Inertia is analogous to mass, so Torque is analogous to force.

The word Torque comes from the Latin word meaning "to twist", and its symbol is the greek letter "τ". Any common examples?
A Torque occurs when a force is applied to a rigid body that is free to rotate about its axis.



$\tau = r_{\perp} F \sin\varphi$
Where r is the radius, F is the applied force, and φ is the angle from the force to the radius.

The torque is often written as:

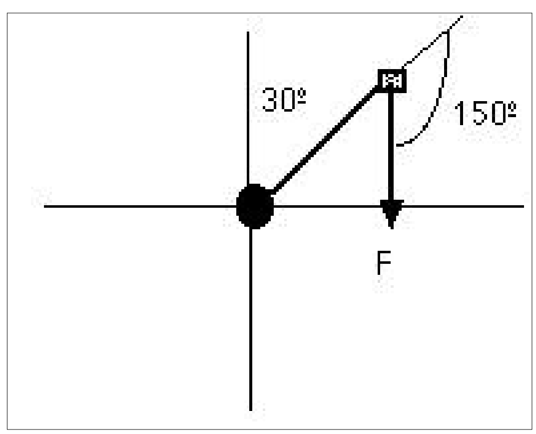
$\tau = r F$ or $\tau = r F_t$

Where $r_\perp$ is the "Moment Arm" and F_t is the tangential force.

The SI Unit for torque is the Newton-Meter

Example:
The length of a bicycle pedal arm is .15 meters. A rider can excerpt a downward force of 100 Newtons.
a) What is the torque when the pedal arm is at 30º from vertical?

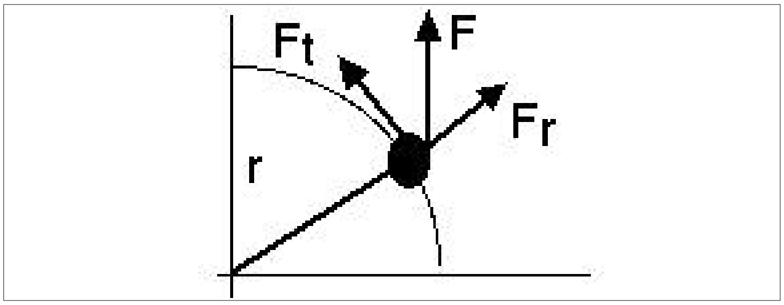
$\perp$



$\tau = r F \sin\varphi$
$\tau = .15(100) \sin 150$
$\tau = 7.5$ N-M
or, $\tau = r F = .15 \sin 30(100) = 7.5$ N-M

b) 90º?

c) 180º?



ISAAC NEWTON RETURNS:

Take a single-particle mass, attach it to the end of a massless rod, and apply a force to move it in a circle.

If we examine the tangential force, Newton's second law applies, and $F_t = ma_t$.

The torque is: $\tau = F_t r$

Substitute for F_t : $\tau = ma_t r$

Remembering that : $a_t = \alpha r$

Substitute for a_t : $\tau = m\alpha r\ r$, or $\tau = m\alpha r^2$

or $\tau = (mr^2)\alpha$

Since Rotational Inertia for a rod is just $I = (mr^2)$

$\tau = I\alpha$ (radian measure)

This holds for any rigid body, and is our angular equivalent for F=ma.

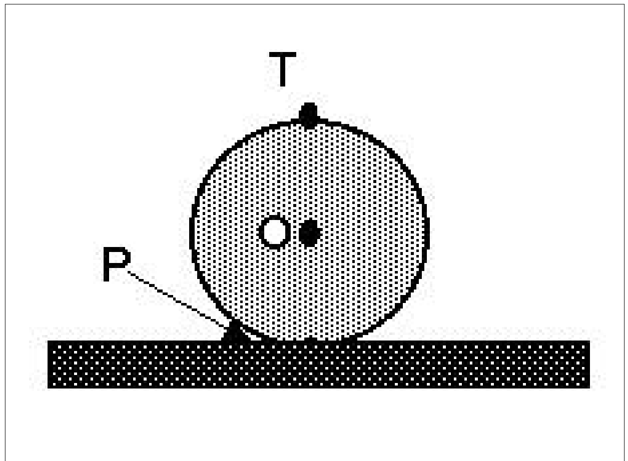
WORK-ENERGY

The Work Energy Theorem is still valid in rotational, and is often useful: $W = \Delta K$

ROLLING ALONG:

A Bicycle, a bowling ball, or any rigid object rolling without a fixed axis can be viewed as a rolling object.

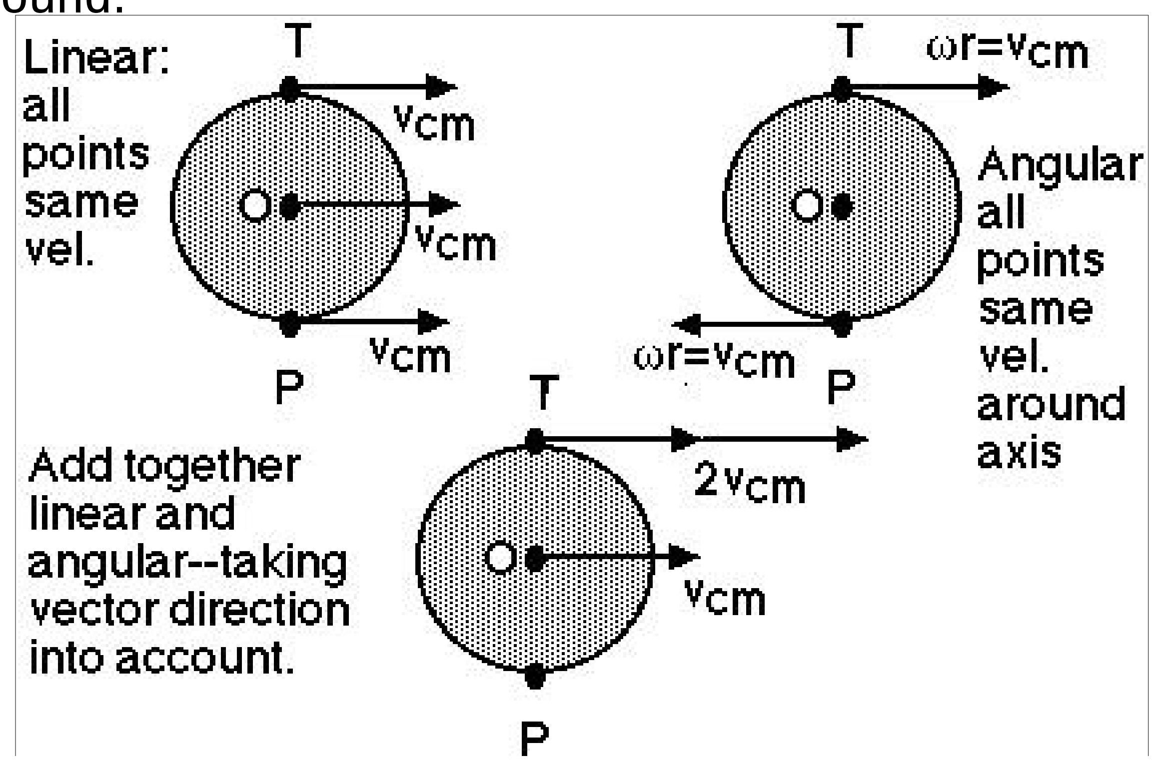
A rolling object can be evaluated as a combination of the translational motion of the center of mass (Vcm) and the rotational motion ($v = \omega r$).



Thus, the top of a rolling wheel moves twice as fast as the center, and the bottom does not move at all.

How?

A wheel can also be viewed as pure rotation about an instantaneous axis at the contact point on the ground.



Using a radius of R, the conversion from angular to linear gives us: $v_{top} = \omega\, 2R$

or $v_{top} = 2\ (\omega R)$

and since ωR is the v_{cm} , $v_{top} = 2v_{cm}$

This is the same result as when we added them together before.

KINETIC ENERGY OF ROLLING:

Choosing the point P as the instantaneous axis,

$K_r = 1/2\ I_p\, \omega^2$

From //Axis Theorem, $I_p = I_{cm} + MR^2$

So, By substitution:

$K_{tot} = 1/2\ I_{cm}\, \omega^2 + 1/2\ MR^2\, \omega^2$

But, as before, $v = \omega r$

So, Finally:

$K_{tot} = 1/2\ I_{cm}\, \omega^2 + 1/2\ MV_{cm}^2$

Is this surprising?

EXAMPLES:

1. A solid Cylinder of mass M=1.4 kg and Radius R=8.5 cm rolls across a table at a speed of 15 cm/s.

a) What is the instantaneous velocity at the top of the cylinder?

$v_{top} = 2v_{cm}$

Twice as much, or 30 cm/s

b) What is the angular speed of the rolling disk?

$v=\omega R$, so $\omega = v/R$ = 15cm/s ÷ 8.5 cm,

= 1.8 rad/s

c) Find the Kinetic Energy:

Use: $K_{tot} = 1/2\ I_{cm}\, \omega^2 + 1/2\ MV_{cm}^2$

and $I_{cm} = 1/2\ MR^2$ and $\omega = v/R$

$K_{tot} = 1/2\ (1/2\ MR^2)\ (v/R)^2 + 1/2\ MV_{cm}^2$

$K_{tot} = 1/4\ MV_{cm}^2 + 1/2\ MV_{cm}^2$

$K_{tot} = 3/4 \ MV_{cm}^2$

$K_{tot} = 3/4 \ (1.4kg)(.15 \ m/s)^2$

$= .024$ J

2. A Bowling Ball of radius 11 cm and M = 7.2 kg rolls down a 2.1 meter ramp inclined at an angle of 34º to the horizontal. How fast is it going at the end of the ramp?

Hint: Use Energy, no friction

$U_{initial} = K_{final}$

$U_{initial} = mgh = Mg(L\sin 34) = Mg(1.17)$

$Mg(1.17) = K_{tot} = 1/2 \ I_{cm} \omega^2 + 1/2 \ MV_{cm}^2$

$I_{cm} = 2/5 \ M R^2$ for a solid sphere, $\omega = v/R$

$Mg(1.17) = 1/2 \ (2/5 \ M R^2)(v_{cm}/R)^2 + 1/2$

MV_{cm}^2

$g(1.17) = 1/2 \ (2/5 \ )(v_{cm})^2 + 1/2 \ V_{cm}^2$

$V_{cm}^2 = (10/7)g(1.17)$, $V_{cm} = 4.1$ m/s

More Linear/Angular Relations

Just as position, velocity and acceleration had their angular equivalents, so do our remaining linear equations. All of these are as we should expect:

TRANSLATIONAL	ROTATIONAL
$W = Fd$	$W = \tau\theta$
$P = Fv$	$P = \tau\omega$
$p = mv$	$L = I\omega$

The Angular Momentum, L

Angular momentum is defined as

$l = r \times p = m(r \times v)$ where l has a direction perpendicular to the r-v plane, assumed the "z" axis.

Thus angular momentum is the tendency of an object to maintain its axis of rotation.

This is the equation for a particle. For a solid object rotating about an axis, we use $L = I\omega$.
This comes from substituting $v = \omega r$, then
$l = mr^2 \omega$, and use "I" for "mr^2"
NEWTON'S LAW
Remembering $\sum F = ma = dp/dt$,

$\sum\tau = dl/dt$ $(=mr\, dv/dt = mra=mar=Fr)$

LAW OF CONSERVATION OF ANGULAR MOMENTUM

If no net external torque acts on a system, the angular momentum of that system remains constant, no matter what changes take place within the system.

EXAMPLES?
-Skater
-Frisbee
-Satellite
-Gyroscope Wheel

PROBLEMS:

1. A Skater is spinning at 1 rps with her arms outstretched with $I = 2.4$ kg-m^2. She pulls in her arms, and I is now 1.2 kg-m^2.

a) What is the new angular velocity?

Use conservation.

$I_1\omega_1 = I_2\omega_2$ thus: $\omega_2 = I_1/I_2\ \omega_1 = 2.4/1.2\ (1) = 2$rps

b) How much work for the skater to pull her arms in?

$W = \Delta K$

change rps to radians, use $K = .5I\omega^2$

W = 95 - 47 = 48 J

2. What is the angular momentum of the earth on its axis?

$L = I\omega = .4MR^2(2\pi/T \text{ rad/s})$

7.1 EE 10^{33} kg-m^2/s or J-s

Direction?

More Problems: Ch 12 16, 43,45,51,55, 56,60